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Sizable D-term contribution as a signature of the $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT model

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We show that the sizable D-term contributions to the sfermion mass spectrum can be signatures of a certain grand unified theory (GUT), $E_6 \times SU(2)_F \times U(1)_A$ GUT. Note that these D-term contributions destroy the degeneracy of sfermion masses among different generations in this model. This is different from previous works, which have argued for the D-term contributions, which destroy the degeneracy of masses only between sfermions with different quantum charges, as a signature of GUT with a larger rank unification group. Such D-terms are strongly constrained by the flavor-changing neutral current processes if the SUSY breaking scale is the weak scale. However, in $E_6 \times SU(2)_F \times U(1)_A$, a natural SUSY-type sfermion mass spectrum is obtained, and if the masses of $\mathbf{10}_3$ sfermions are larger than $O(1\,\mathrm{TeV})$ to realize the 126 GeV Higgs and the other sfermion masses are $O(10\,\mathrm{TeV})$, then a sizable D-term contribution is allowed. If these D-terms can be observed in future experiments, like the 100 TeV proton collider or muon collider, we may confirm the $E_6 \times SU(2)_F \times U(1)_A$ GUT.

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1. Introduction

Grand unified theory (GUT) [1] is one of the most promising extensions of the standard model (SM). It unifies not only three gauge interactions in the SM into a single gauge interaction, but also, e.g., quarks and leptons into a few multiplets, $\mathbf{10}$ and $\bar{\mathbf{5}}$ of $SU(5)$. Moreover, there is experimental support for both unifications. For the unification of forces, the measured values of three gauge couplings are quantitatively consistent with the unification of gauge interactions in the minimal supersymmetric (SUSY) SM (MSSM). For the unification of matters in $SU(5)$ GUT, if we assume that the $\mathbf{10}$ matter fields induce stronger hierarchies of Yukawa couplings than the $\bar{\mathbf{5}}$ matter fields, the various measured hierarchies of quark and lepton masses and mixings can be explained qualitatively at the same time [2–9].

In E_6 unification [10–17], the above assumption for the origin of the hierarchies can be derived [9]. As a result, we can obtain various realistic hierarchies of Yukawa couplings from one basic Yukawa hierarchy that realizes the hierarchy of up-type quarks. Moreover, if the family symmetry [18–27], $SU(3)_F$ or $SU(2)_F$, is introduced, we can obtain a model in which all three generations of quarks and leptons can be unified into a single multiplet or two multiplets, and after breaking the family symmetry and E_6 unified symmetry, realistic quark and lepton masses and mixings can be realized [28–31]. Such models predict a peculiar sfermion mass spectrum in which all sfermions except the

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third generation of the **10** matter $\mathbf{10}_3$ have universal sfermion masses. This is called modified universal sfermion masses (MUSM). When the mass of $\mathbf{10}_3$ is smaller than the other universal sfermion masses, the mass spectrum is nothing but the natural SUSY-type sfermion mass spectrum [32,33], in which the SUSY flavor-changing neutral current (FCNC) processes are suppressed due to large sfermion masses while the weak scale is stabilized.

The most difficult problem in the SUSY GUT scenario is the doublet–triplet splitting problem (for a review, see Ref. [34]). One pair of Higgs doublets in the MSSM can be included in $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ with one pair of triplet (colored) Higgs. The mass of the triplet Higgs must be larger than the GUT scale to stabilize the nucleon, while the mass of the doublet Higgs must be around the weak scale. It is difficult to realize such a splitting without fine-tuning. Several ideas to solve this problem have been discussed in various models in the literature. Unfortunately, in most of the models, very small parameters are required or the terms that are allowed by the symmetry are dropped just by hand. Such a feature is, in a sense, fine-tuning.

If the anomalous $U(1)_A$ gauge symmetry [35–38] is introduced, the doublet–triplet splitting problem can be solved in a natural assumption that all the interactions that are allowed by the symmetry are introduced with $O(1)$ coefficients. Note that all higher-dimensional interactions that are allowed by the symmetry are introduced. Because of this natural assumption, we call the GUT scenario with the anomalous $U(1)_A$ gauge symmetry "natural GUT" [4–8,39]. Note that in natural GUT the vacuum expectation value (VEV) of an operator O_i that is a singlet under gauge groups, except $U(1)_A$, can be determined by its $U(1)_A$ charge o_i as

$$\langle O_i \rangle = \begin{cases} 0 & (o_i > 0) \\ \lambda^{-o_i} & (o_i \leq 0) \end{cases}, \tag{1}$$

where λ is determined from the Fayet–Iliopoulos parameter ξ as $\lambda \equiv \xi/\Lambda$. In this paper, we take $\lambda \sim 0.22$ and adopt the unit in which the cutoff $\Lambda = 1$. This feature is important in solving the doublet–triplet splitting problem.

If we consider the E_6 GUT with family symmetry and the anomalous $U(1)_A$ gauge symmetry at the same time, more attractive GUT model with $E_6 \times SU(2)_F \times U(1)_A$ gauge symmetry can be obtained. Since the μ problem is also solved in the natural GUT [39,40], we can discuss the SUSY CP problem. Actually, by imposing the CP symmetry and considering the spontaneous CP violation, we can solve not only the usual SUSY CP problem but also the new SUSY CP problem on the chromoelectric dipole moment (CEDM) [41–43], which is more serious in the natural SUSY-type sfermion mass spectrum [44–47].

How can this interesting SUSY GUT scenario be tested? Since the unification scale is so large that it is difficult to produce GUT particles directly, it is important to examine various indirect searches. The most promising candidate for the indirect search is to find the nucleon decay. In the natural GUT, the nucleon decay via dimension-6 operators is enhanced while the nucleon decay via dimension-5 operators is suppressed [4–8]. We have proposed how to identify the unification group in the natural GUT by observing the decay modes of nucleons in Refs. [48,49].

Recently, one of the authors has pointed out that if the gravitino mass is $O(100\,\text{TeV})$ to solve the gravitino problem and the other SUSY breaking parameters are $O(1\,\text{TeV})$ for the gauge hierarchy problem, the little hierarchy problem becomes less severe ($O(\%)$ tuning is realized) [50]. In this scenario, a sizable anomaly mediation [51,52] contribution cancels the renormalization group (RG) effects of the gravity mediation. As a result, if the mirage scale, at which three gaugino masses meet, is $O(\text{TeV})$, we can observe directly the gravity contribution at the GUT scale by low-energy

Table 1. Field contents of matters and Higgs and charge assignments under $E_6 \times SU(2)_F \times U(1)_A \times Z_6$.

	Ψ_a	Ψ_3	F_a	$\bar{F}^a$	Φ	$\bar{\Phi}$	C	$\bar{C}$	A	Z_3	Θ
E_6	**27**	**27**	**1**	**1**	**27**	$\overline{\mathbf{27}}$	**27**	$\overline{\mathbf{27}}$	**78**	**1**	**1**
$SU(2)_F$	**2**	**1**	**2**	$\bar{\mathbf{2}}$	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$U(1)_A$	4	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{5}{2}$	-3	1	-4	-1	$-\frac{1}{2}$	$-\frac{3}{2}$	-1
Z_6	3	3	1	0	0	0	5	0	0	0	0

experiments at the TeV scale, except for the stop, the right-handed stau, and the up-type Higgs masses. (In the usual mirage mediation scenario, all sfermion masses can converge at the mirage scale by taking the special boundary conditions [53–57], while in the scenario without the special conditions the stop masses do not become the gravity contribution at the GUT scale because of the large top Yukawa couplings [50].)

Note that the sfermions' masses other than the stops and the right-handed stau do not have to be universal in the arguments [50]. Therefore, we can test the GUT scenario by measuring the sfermion mass spectrum if some signatures of the GUT appear in the sfermion masses. For example, if the rank of the unification group is larger than 4, the non-vanishing D-term contributions, which are usually flavor blind, can be a signature of the GUT scenario [58–61]. The MUSM can be a signature of $E_6 \times SU(2)_F$ GUT, since the most serious CEDM constraints for the natural SUSY-type mass spectrum can be avoided in the scenario by spontaneous CP violation [44–46]. One more interesting test for the $E_6 \times SU(2)_F$ GUT scenario is to observe the non-vanishing D-term contributions of the E_6 and $SU(2)_F$ gauge symmetry to sfermion masses. This is interesting because they spoil the universality of the sfermion masses. Before the LHC found the 126 GeV Higgs [62,63], these D-term contributions were strongly constrained to be small from the various FCNC processes [64]. However, the stop mass must be larger than 1 TeV in order to realize the 126 GeV Higgs, and therefore the other sfermion masses can be $O(10\ \text{TeV})$. Since FCNC constraints become much weaker when the SUSY breaking scale is $O(10\ \text{TeV})$, a sizable D-term contribution may be allowed.

In this paper, we clarify the D-term contributions of the $E_6 \times SU(2)_F \times U(1)_A$ GUT model and discuss the FCNC constraint from ϵ of $K^0\bar{K}^0$ mixing because it is the strongest. We will conclude that a sizable D-term contribution is possible. If the D-term contributions are sufficiently large and observed by future experiments, e.g., by the SuperLHC, then we can obtain precious evidence of the GUT scenario.

2. $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT model

In this section we give a short review of the $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT model. Please see Refs. [10–17] for a more detailed explanation of the model. The notation for the GUT model in this paper is almost the same as that for the model in Ref. [49].

2.1. *Yukawa matrices for quarks and charged leptons*

The contents of matters and Higgs and their charge assignments are shown in Table 1, though this model is just an example. In the model we introduce three 27 dimensional (fundamental) fields of E_6 as matters. The **27** is decomposed in the $E_6 \supset SO(10) \times U(1)_{V'}$ notation (and in the $[SO(10) \supset SU(5) \times U(1)_V]$ notation) as

$$\mathbf{27} = \mathbf{16}_1[\mathbf{10}_1 + \bar{\mathbf{5}}_{-3} + \mathbf{1}_5] + \mathbf{10}_{-2}[\mathbf{5}_{-2} + \bar{\mathbf{5}}'_2] + \mathbf{1}'_4[\mathbf{1}'_0]. \tag{2}$$

The **27** of E_6 includes not only spinor **16** but also vector **10** of $SO(10)$. These **10**s of $SO(10)$ play an important role in obtaining realistic quark and lepton masses and mixings. The spinor and vector of $SO(10)$ are decomposed in the $SU(3)_C \times SU(2)_L \times U(1)_Y$ notation as

$$\mathbf{16} \to \underbrace{q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1}_{\mathbf{10}} + \underbrace{d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{\nu_R^c(\mathbf{1},\mathbf{1})_0}_{\mathbf{1}}, \tag{3}$$

$$\mathbf{10} \to \underbrace{D_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + L_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}'} + \underbrace{\overline{D_R^c}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}} + \overline{L_L}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}}_{\mathbf{5}}. \tag{4}$$

Matter fields $\mathbf{27}_i$ $(i = 1, 2, 3)$ include six $\bar{\mathbf{5}}$s of $SU(5)$. Three of the six $\bar{\mathbf{5}}$s become superheavy by developing the VEVs $\langle\Phi\rangle$, which breaks E_6 into $SO(10)$, and $\langle C\rangle$, which breaks $SO(10)$ into $SU(5)$, through the superpotential

$$\begin{aligned} W_Y = &\left(a\Psi_3\Psi_3 + b\Psi_3\bar{F}^a\Psi_a + c\bar{F}^a\Psi_a\bar{F}^b\Psi_b\right)\Phi + d\left(\Psi_a, \Phi, \bar{\Phi}, A, Z_3, \Theta\right) \\ &+ f'\bar{F}^a\Psi_a\epsilon^{bc}F_b\Psi_c C + g'\Psi_3\epsilon^{ab}F_a\Psi_b C, \end{aligned} \tag{5}$$

where a, b, c, f', and g' are $O(1)$ coefficients. $d(\Psi_a, \Phi, \bar{\Phi}, A, Z_3, \Theta)$ is a gauge-invariant function of $\Psi_a, \Phi, \bar{\Phi}, A, Z_3$, and Θ, and it contributes to $\Psi_1\Psi_2\Phi$. The other three $\bar{\mathbf{5}}$s become the SM $\bar{\mathbf{5}}_i^0$ whose main components become $(\bar{\mathbf{5}}_1^0, \bar{\mathbf{5}}_2^0, \bar{\mathbf{5}}_3^0) \sim (\bar{\mathbf{5}}_1, \bar{\mathbf{5}}'_1, \bar{\mathbf{5}}_2)$. This is a critical observation in calculating the D-term contribution to the sfermion masses.

After developing VEVs $\langle\bar{\Phi}\Phi\rangle \sim \lambda^2$, $\langle\bar{C}C\rangle \sim \lambda^5$, $\langle A\rangle \sim \lambda^{1/2}$, $\langle\bar{F}\rangle \sim (0, \lambda^2)$, and $\langle F\rangle \sim (0, e^{i\rho}\lambda^2)$[1], we can obtain the up-type Yukawa matrix Y_u, down-type Yukawa matrix Y_d, and charged lepton Yukawa matrix Y_e at the GUT scale as

$$Y_u = \begin{pmatrix} 0 & \frac{1}{3}d_q\lambda^5 & 0 \\ -\frac{1}{3}d_q\lambda^5 & c\lambda^4 & b\lambda^2 \\ 0 & b\lambda^2 & a \end{pmatrix}, \tag{6}$$

$$Y_d = \begin{pmatrix} -\left(\frac{(bg-af)^2}{ac-b^2} + g^2\right)\frac{\beta_H}{a}e^{i(2\rho-\delta)}\lambda^6 & -\frac{bg-af}{ac-b^2}\frac{2}{3}d_5\beta_H e^{i(\rho-\delta)}\lambda^{5.5} & \frac{1}{3}d_q\lambda^5 \\ \left(-\frac{d_q}{3} - \frac{bg-af}{ac-b^2}\frac{b\frac{2}{3}d_5}{g}\right)\lambda^5 & \left(f\beta_H e^{i(\rho-\delta)} - \frac{\left(\frac{2}{3}d_5\right)^2}{ac-b^2}\frac{ab}{g}e^{-i\rho}\right)\lambda^{4.5} & \frac{cg-bf}{g}\lambda^4 \\ -\frac{bg-af}{ac-b^2}\frac{a\frac{2}{3}d_5}{g}\lambda^3 & \left(g\beta_H e^{i(\rho-\delta)} - \frac{\left(\frac{2}{3}d_5\right)^2}{ac-b^2}\frac{a^2}{g}e^{-i\rho}\right)\lambda^{2.5} & \frac{bg-af}{g}\lambda^2 \end{pmatrix}, \tag{7}$$

$$Y_e = \begin{pmatrix} -\left(\frac{(bg-af)^2}{ac-b^2} + g^2\right)\frac{\beta_H}{a}e^{i(2\rho-\delta)}\lambda^6 & d_l\lambda^5 & 0 \\ 0 & f\beta_H e^{i(\rho-\delta)}\lambda^{4.5} & g\beta_H e^{i(\rho-\delta)}\lambda^{2.5} \\ -d_l\lambda^5 & \frac{cg-bf}{g}\lambda^4 & \frac{bg-af}{g}\lambda^2 \end{pmatrix}, \tag{8}$$

[1] These VEVs of F and $\bar{F}$ are consistent with the relations (1) because F and $\bar{F}$ are not singlet under $SU(2)_F$. Actually, the $SU(2)_F$ singlet operator $\bar{F}F$ satisfies the relation (1). These VEVs are determined by the D-flatness conditions $|\langle F\rangle| = |\langle\bar{F}\rangle|$.

where a, b, c, d_q, d_5, d_l, f, g, and β_H are real $O(1)$ coefficients, ρ and δ are $O(1)$ phases, and $\lambda \sim 0.22$ is taken to be the Cabibbo angle. In this paper, we begin our arguments from these Yukawa matrices that have only 9 real parameters and 2 CP phases.

2.2. *Mass spectrum of sfermions*

The sfermion mass matrices can be obtained mainly from the SUSY breaking potential

$$\begin{aligned} V_{SB} = {} & m_0^2 |\Psi_a|^2 + m_3^2 |\Psi_3|^2 + m_{11}^2 |\epsilon^{ab}\Psi_a F_b|^2 + m_{22}^2 |\Psi_a \bar{F}^a|^2 \\ & + \left(m_{23}^2 \Psi_3^\dagger \Psi_a \bar{F}^a + m_{13}^2 \lambda^5 \Psi_3^\dagger \epsilon^{ab} \Psi_a \bar{F}_b^\dagger + m_{12}^2 \lambda^5 \left(\Psi_a \bar{F}^a\right)^\dagger \epsilon^{bc} \Psi_b \bar{F}_c^\dagger + \text{h.c.}\right) \\ & + m^2 \left(\Phi^\dagger \Psi^{a\dagger} \Psi_a \Phi\right) + \left(m_{12}'^2 \lambda^2 \bar{C} |\Psi_a|^2 \bar{\Phi}^\dagger + m_{23}'^2 \lambda^2 \bar{C} \Psi_3^\dagger \Psi_a \bar{F}^a \bar{\Phi}^\dagger + \text{h.c.}\right), \end{aligned} \tag{9}$$

where the terms in the last line give the mass terms between $\bar{\mathbf{5}}$ and $\bar{\mathbf{5}}'$ after developing the VEVs $\langle\Phi\rangle$, $\langle\bar{\Phi}\rangle$, $\langle C\rangle$, and $\langle\bar{C}\rangle$. These mass parameters are considered to be the SUSY breaking scale O(1–10 TeV). The D-term contributions are written as

$$\Delta\tilde{m}_\psi^2 = \sum_I Q_I(\psi) D_I, \tag{10}$$

where D_I is the squared gauge coupling times the D-term of $U(1)_{V'}(I=6)$, $U(1)_V(I=10)$, $U(1)_F(I=F)$, and $U(1)_A(I=A)$, and $Q_I(\psi)$ is the $U(1)$ charge of the field ψ. Here $U(1)_F$ is the Cartan part of $SU(2)_F$. As a result, the sparticle masses for $\bar{\mathbf{5}}$, $\bar{\mathbf{5}}'$, and $\mathbf{10}$ of $SU(5)$ are

$$\begin{aligned} \tilde{m}_{\bar{5}}^2 = {} & \begin{pmatrix} m_0^2 + \lambda^4 m_{11}^2 & \lambda^9 m_{12}^2 & \lambda^7 m_{13}^2 \\ \lambda^9 m_{12}^2 & m_0^2 + \lambda^4 m_{22}^2 & \lambda^2 m_{23}^2 \\ \lambda^7 m_{13}^2 & \lambda^2 m_{23}^2 & m_3^2 \end{pmatrix} + D_6 \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \\ & + D_{10} \begin{pmatrix} -3 & & \\ & -3 & \\ & & -3 \end{pmatrix} + D_F \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} + D_A \begin{pmatrix} 4 & & \\ & 4 & \\ & & \frac{3}{2} \end{pmatrix}, \end{aligned} \tag{11}$$

$$\begin{aligned} \tilde{m}_{\bar{5}'}^2 = {} & \begin{pmatrix} m_0^2 + \lambda^2 m^2 + \lambda^4 m_{11}^2 & \lambda^9 m_{12}^2 & \lambda^7 m_{13}^2 \\ \lambda^9 m_{12}^2 & m_0^2 + \lambda^2 m^2 + \lambda^4 m_{22}^2 & \lambda^2 m_{23}^2 \\ \lambda^7 m_{13}^2 & \lambda^2 m_{23}^2 & m_3^2 \end{pmatrix} \\ & + D_6 \begin{pmatrix} -2 & & \\ & -2 & \\ & & -2 \end{pmatrix} + D_{10} \begin{pmatrix} 2 & & \\ & 2 & \\ & & 2 \end{pmatrix} + D_F \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \\ & + D_A \begin{pmatrix} 4 & & \\ & 4 & \\ & & \frac{3}{2} \end{pmatrix}, \end{aligned} \tag{12}$$

$$\begin{aligned} \tilde{m}_{10}^2 = {} & \begin{pmatrix} m_0^2 + \lambda^4 m_{11}^2 & \lambda^9 m_{12}^2 & \lambda^7 m_{13}^2 \\ \lambda^9 m_{12}^2 & m_0^2 + \lambda^4 m_{22}^2 & \lambda^2 m_{23}^2 \\ \lambda^7 m_{13}^2 & \lambda^2 m_{23}^2 & m_3^2 \end{pmatrix} + D_6 \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \\ & + D_{10} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + D_F \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} + D_A \begin{pmatrix} 4 & & \\ & 4 & \\ & & \frac{3}{2} \end{pmatrix}, \end{aligned} \tag{13}$$

where the contribution of the term $m^2\Phi^\dagger\Psi^{a\dagger}\Psi_a\Phi$ to $|\mathbf{16}_{\Psi_a}|^2$ is included in m_0^2 by redefinition of m_0^2. Then, the sfermion mass matrix for SM $\bar{\mathbf{5}}$ fields, which are represented as $(\bar{\mathbf{5}}_1^0, \bar{\mathbf{5}}_2^0, \bar{\mathbf{5}}_3^0) \sim (\bar{\mathbf{5}}_1, \bar{\mathbf{5}}'_1, \bar{\mathbf{5}}_2)$, becomes

$$\begin{aligned}\tilde{m}_{\bar{5}^0}^2 \sim & \begin{pmatrix} m_0^2+\lambda^4 m_{11}^2 & \lambda^{5.5}m_{12}'^2 & \lambda^9 m_{12}^2 \\ \lambda^{5.5}m_{12}'^2 & m_0^2+\lambda^2m^2+\lambda^4 m_{11}^2 & \lambda^{7.5}m_{23}'^2 \\ \lambda^9 m_{12}^2 & \lambda^{7.5}m_{23}'^2 & m_0^2+\lambda^4 m_{22}^2 \end{pmatrix} \\ & + D_6\begin{pmatrix} 1 & & \\ & -2 & \\ & & 1 \end{pmatrix} + D_{10}\begin{pmatrix} -3 & & \\ & 2 & \\ & & -3 \end{pmatrix} \\ & + D_F\begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} + D_A\begin{pmatrix} 4 & & \\ & 4 & \\ & & 4 \end{pmatrix}. \end{aligned} \tag{14}$$

Moreover, the contributions from the sub-leading components of $\bar{\mathbf{5}}_i^0$ become

$$\Delta\tilde{m}_{\bar{5}^0}^2 \sim (m_0^2 - m_3^2)\begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^{5.5} & \lambda^5 & \lambda^{4.5} \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \end{pmatrix}. \tag{15}$$

These sfermion mass matrices give interesting predictions of $E_6 \times SU(2)_F \times U(1)_A$ GUT, though the terms that are suppressed by the power of λ are strongly dependent on the explicit model. In the next section, we discuss how to obtain GUT information from the sfermion mass spectrum.

3. Signatures of $E_6 \times SU(2)_F \times U(1)_A$ GUT from the sfermion mass spectrum

Suppose that, in the future, all sfermion and gaugino masses are measured by experiments, and three gaugino masses meet at the GUT scale or at a mirage scale. Then, in principle, we can calculate the sfermion mass spectrum at the GUT scale or the mirage scale from the measured values by RG equations. If the mass spectrum respects the $SU(5)$ symmetry, this can be a signature for GUT, though in the generalized mirage mediation scenario, sfermions that have large Yukawa couplings like top Yukawa coupling do not have masses that are consistent with the $SU(5)$ GUT symmetry generically at the mirage scale [50]. In this section, we discuss the signatures of GUT in the sfermion mass spectrum at the GUT scale. The constraints from the FCNC processes will be discussed in the next section.

If the observed sfermion mass spectrum at the GUT scale or at the mirage scale is the MUSM as

$$\tilde{m}_{10}^2 \sim \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_3^2 \end{pmatrix}, \quad \tilde{m}_{\bar{5}^0}^2 \sim \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_0^2 \end{pmatrix}, \tag{16}$$

$E_6 \times SU(2)_F$ GUT is strongly implied. (The third generation of $\mathbf{10}$ of $SU(5)$ may not respect $SU(5)$ in generalized mirage mediation because of large top Yukawa coupling [50].) Of course, the MUSM is nothing but a natural SUSY-type sfermion mass spectrum, which is predicted by a lot of models. However, the generically natural SUSY-type sfermion mass spectrum suffers from the CEDM problem [41–43], and there are few models in which the problem can be solved in a natural way. We would like to emphasize that the CEDM problem can be solved in the $E_6 \times SU(2)_F \times U(1)_A$ GUT by spontaneous CP violation in a non-trivial way [44–46].

In order to obtain more specific signatures of the $E_6 \times SU(2)_F \times U(1)_A$ GUT, we study the D-term contributions. For a while, we neglect the terms that are suppressed by the power of λ. We will discuss these terms later. Then, the mass matrices of $\tilde{m}^2_{10}$ and $\tilde{m}^2_{\bar{5}0}$ are rewritten as

$$\tilde{m}^2_{10} = (m_0^2 + D_6 + D_{10} + D_F + 4D_A)\mathbf{1}_{3\times 3} + \begin{pmatrix} 0 & & \\ & -2D_F & \\ & & -D_F - \frac{5}{2}D_A + m_3^2 - m_0^2 \end{pmatrix}$$
$$\equiv m^2_{10,0}\mathbf{1}_{3\times 3} + \begin{pmatrix} 0 & & \\ & \Delta m^2_{10,2} & \\ & & \Delta m^2_{10,3} \end{pmatrix}, \tag{17}$$

$$\tilde{m}^2_{\bar{5}0} = (m_0^2 + D_6 - 3D_{10} + D_F + 4D_A)\mathbf{1}_{3\times 3} + \begin{pmatrix} 0 & & \\ & -3D_6 + 5D_{10} & \\ & & -2D_F \end{pmatrix}$$
$$\equiv m^2_{\bar{5}0,0}\mathbf{1}_{3\times 3} + \begin{pmatrix} 0 & & \\ & \Delta m^2_{\bar{5},2} & \\ & & \Delta m^2_{\bar{5},3} \end{pmatrix}, \tag{18}$$

where $\mathbf{1}_{3\times 3}$ is a 3×3 unit matrix. A non-trivial prediction of this model is $\Delta m^2_{10,2} = \Delta m^2_{\bar{5},3}$. If this relation is observed, we obtain strong evidence for this model and can know the D_F. The D_6 and D_{10} can be determined if $m^2_{10,0} - m^2_{\bar{5}0,0}$ and $\Delta m^2_{\bar{5},2}$ are observed. If these small modifications from the MUSM and $\Delta m^2_{10,2} = \Delta m^2_{\bar{5},3}$ are observed, we think that the $E_6 \times SU(2)_F \times U(1)_A$ model can be established.

What size D-terms are allowed? If these D-terms are very small, it may become difficult to measure them, and if these D-terms are large, the FCNC constraints cannot be satisfied. In the next section, we study the constraints to the D-terms from the FCNC processes, especially from the ϵ parameter in $K^0\bar{K}^0$ mixing, which gives the strongest constraints.

4. FCNC constraints to *D*-terms

In this section, we focus on the natural SUSY-type sfermion masses, i.e., $m_0 \gg m_3$, because the FCNC constraints become weaker and a sizable D-term may be allowed. Therefore, we fix $\Delta m^2_{10,3} = m_0^2$. To obtain the 126 GeV Higgs, m_3 must be larger than 1 TeV. Since the smaller m_3 tends to be more natural, we take $m_3 \sim O(1\,\text{TeV})$. In the literature, the upper bound for the ratio m_0/m_3 has been studied; it is derived from the requirement of the positivity of the stop mass square to be roughly 5 through a two-loop RG contribution [32,33]. Therefore, we expect that m_0 is $O(10\,\text{TeV})$. In this paper, we do not argue the upper bound of m_0/m_3 explicitly, because a larger stop mass can always satisfy the positivity and the upper bound is dependent on the explicit models between the GUT scale and the SUSY breaking scale.

If the D-term contributions can be negligible, the contributions to the FCNC processes from the MUSM become sufficiently small to satisfy the experimental bounds [28–31], though the CEDM constraint is quite severe, which will be discussed later. When the D-terms become sizable, the strongest constraints can be given from the CP-violating parameter ϵ in $K^0\bar{K}^0$ mixing. Basically, if these constraints are satisfied, the other FCNC constraints are automatically satisfied. Therefore, we consider here the constraints from the CP-violating parameter ϵ in $K^0\bar{K}^0$ mixing.

Since we calculate constraints from the FCNC processes with the mass eigenstates of quarks and leptons, we need diagonalizing matrices that make Yukawa matrices diagonal as

$$\psi_{Li}(Y_\psi)_{ij}\psi^c_{Rj} = \left(L^\dagger_\psi \psi_L\right)_i \left(L^T_\psi Y_\psi R_\psi\right)_{ij} \left(R^\dagger_\psi \psi^c_R\right)_j \equiv \psi'_{Li}\left(Y^D_\psi\right)_{ij}\psi'^c_{Rj}, \tag{19}$$

where ψ is a flavor eigenstate, ψ' is a mass eigenstate, and Y^D_ψ is a diagonalized matrix of ψ. We summarize the detailed expression of these diagonalizing matrices with the explicit $O(1)$ coefficients in Appendix A. Here we roughly show the diagonalizing matrices for up-type quark, down-type quark, and charged lepton without $O(1)$ coefficients as

$$L_u \sim \begin{pmatrix} 1 & \frac{1}{3}\lambda & 0 \\ \frac{1}{3}\lambda & 1 & \lambda^2 \\ \frac{1}{3}\lambda^3 & \lambda^2 & 1 \end{pmatrix},\ R_u \sim \begin{pmatrix} 1 & \frac{1}{3}\lambda & 0 \\ \frac{1}{3}\lambda & 1 & \lambda^2 \\ \frac{1}{3}\lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{20}$$

$$L_d \sim \begin{pmatrix} 1 & \left(\frac{2}{3}+i\frac{4}{27}\right)\lambda & \frac{1}{3}\lambda^3 \\ \left(\frac{2}{3}+i\frac{4}{27}\right)\lambda & 1 & \lambda^2 \\ \left(\frac{2}{3}+i\frac{4}{27}\right)\lambda^3 & \lambda^2 & 1 \end{pmatrix},\ R_d \sim \begin{pmatrix} 1 & \frac{2}{3}(1+i)\lambda^{0.5} & \frac{2}{3}\lambda \\ \frac{2}{3}(1+i)\lambda^{0.5} & 1 & (1+i)\lambda^{0.5} \\ \frac{2}{3}(1+i)\lambda & (1+i)\lambda^{0.5} & 1 \end{pmatrix}, \tag{21}$$

$$L_e \sim \begin{pmatrix} 1 & (1+i)\lambda^{0.5} & 0 \\ (1+i)\lambda^{0.5} & 1 & (1+i)\lambda^{0.5} \\ \lambda & (1+i)\lambda^{0.5} & 1 \end{pmatrix},\ R_e \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{22}$$

$$L_\nu \sim \begin{pmatrix} 1 & (1+i)\lambda^{0.5} & (1+i)\lambda \\ (1+i)\lambda^{0.5} & 1 & (1+i)\lambda^{0.5} \\ (1+i)\lambda & (1+i)\lambda^{0.5} & 1 \end{pmatrix}. \tag{23}$$

We have two types of diagonalizing matrices for **10** of $SU(5)$ sfermions and for $\bar{\mathbf{5}}$ sfermions as

$$U_{\mathrm{CKM-type}} \equiv \begin{pmatrix} 1 & a_{12}\lambda & a_{13}\lambda^3 \\ a_{21}\lambda & 1 & a_{23}\lambda^2 \\ a_{31}\lambda^3 & a_{32}\lambda^2 & 1 \end{pmatrix} \quad (\text{for } L_u, L_d, R_u, \text{ and } R_e) \tag{24}$$

$$U_{\mathrm{MNS-type}} \equiv \begin{pmatrix} b_{11} & b_{12}\lambda^{0.5} & b_{13}\lambda \\ b_{21}\lambda^{0.5} & b_{22} & b_{23}\lambda^{0.5} \\ b_{31}\lambda & b_{32}\lambda^{0.5} & b_{33} \end{pmatrix} \quad (\text{for } L_e, L_\nu, \text{ and } R_d), \tag{25}$$

where a_{ij} and b_{ij} are generically complex $O(1)$ coefficients, respectively. The mass insertion parameters defined as

$$(\delta^\psi_{ij})_{\Gamma\Gamma} \equiv \frac{(\Gamma^\dagger_\psi \tilde{m}^2_{\psi_\Gamma}\Gamma_\psi)_{ij}}{m^2_{\tilde{\psi}}} \quad (\Gamma = L, R), \tag{26}$$

where $m_{\tilde{\psi}}$ is the averaged mass of $\psi = u, d, e, \nu$ and is taken as m_0 in many cases in this paper, can be calculated as

$$(\delta_{ij}^{\psi})_{\Gamma\Gamma} = \begin{pmatrix} \cdots & a_{21}^* \lambda \Delta m_{10,2}^2 + a_{31}^* a_{32} \lambda^5 \Delta m_{10,3}^2 & (a_{21}^* a_{23} \Delta m_{10\ 2}^2 + a_{31}^* \Delta m_{10,3}^2)\lambda^3 \\ \cdots & \cdots & (a_{23} \Delta m_{10,2}^2 + a_{32}^* \Delta m_{10,3}^2)\lambda^2 \\ \cdots & \cdots & \cdots \end{pmatrix} \Bigg/ m_{\tilde{\psi}}^2, \tag{27}$$

$$(\delta_{ij}^{\psi})_{\Gamma\Gamma} = \begin{pmatrix} \cdots & b_{21}^* b_{22} \lambda^{0.5} \Delta m_{\bar{5},2}^2 + b_{31}^* b_{32} \lambda^{1.5} \Delta m_{\bar{5},3}^2 & (b_{21}^* b_{23} \Delta m_{\bar{5},2}^2 + b_{31}^* b_{33} \Delta m_{\bar{5},3}^2)\lambda \\ \cdots & \cdots & (b_{22}^* b_{23} \Delta m_{\bar{5},2}^2 + b_{32}^* b_{33} \Delta m_{\bar{5},3}^2)\lambda^{0.5} \\ \cdots & \cdots & \cdots \end{pmatrix} \Bigg/ m_{\tilde{\psi}}^2, \tag{28}$$

for **10** fields and $\bar{\mathbf{5}}$ fields, respectively. In Appendix B, we show each mass insertion in this model with explicit $O(1)$ coefficients.

Let us calculate the constraints from the ϵ parameter in $K^0\bar{K}^0$ mixing. We use the constraints for $(\delta_{12}^d)_{LL}$ and $(\delta_{12}^d)_{RR}$ as

$$\sqrt{|\mathrm{Im}(\delta_{12}^d)_{LL}^2|} < 2.9 \times 10^{-3} \left(\frac{m_{\tilde{d}}}{500\,\mathrm{GeV}}\right), \tag{29}$$

$$\sqrt{|\mathrm{Im}(\delta_{12}^d)_{RR}^2|} < 2.9 \times 10^{-3} \left(\frac{m_{\tilde{d}}}{500\,\mathrm{GeV}}\right), \tag{30}$$

$$\sqrt{|\mathrm{Im}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}|} < 1.1 \times 10^{-4} \left(\frac{m_{\tilde{d}}}{500\,\mathrm{GeV}}\right), \tag{31}$$

which are obtained in Refs. [65,66] by including the SM contribution and next-to-leading order calculation of QCD. These parameters can roughly be calculated as

$$(\delta_{12}^d)_{LL} \sim \left(\frac{2}{3} + i\frac{4}{27}\right)\left(\lambda \frac{\Delta m_{10,2}^2}{m_{\tilde{d}}^2} + \lambda^5 \frac{\Delta m_{10,3}^2}{m_{\tilde{d}}^2}\right) \tag{32}$$

$$(\delta_{12}^d)_{RR} \sim \frac{2}{3}(1+i)\left(\lambda^{0.5} \frac{\Delta m_{\bar{5},2}^2}{m_{\tilde{d}}^2} + \lambda^{1.5} \frac{\Delta m_{\bar{5},3}^2}{m_{\tilde{d}}^2}\right). \tag{33}$$

By taking $\Delta m_{10,3}^2 = m_0^2 = m_{\tilde{d}}^2$, we can obtain the allowed region in $\left(\sqrt{|\Delta m_{\bar{5},2}^2|}\Big/ m_{\tilde{d}}, \sqrt{|\Delta m_{10,2}^2|}\Big/ m_{\tilde{d}} = \sqrt{|\Delta m_{\bar{5},3}^2|}\Big/ m_{\tilde{d}}\right)$ space, which is shown in Fig. 1. Note that $\Delta m_{10,2}^2 = \Delta m_{\bar{5},3}^2$ is one of the predictions in the $E_6 \times SU(2)_F \times U(1)_A$ model. Roughly, if m_0 is $O(10\,\mathrm{TeV})$, $\sqrt{|\Delta m^2|}$ is allowed to be $O(1\,\mathrm{TeV})$, which is nothing but the scale of m_3. The constraint to $\Delta m_{10,2}^2 = \Delta m_{\bar{5},3}^2$ is stronger, because this contributes to $(\delta_{12}^d)_{LL}$ and $(\delta_{12}^d)_{RR}$ at the same time. On the other hand, the constraint to $\Delta m_{\bar{5},2}^2$ is weaker, and sizable $\Delta m_{\bar{5},2}^2$ can be allowed. Since the E-twisting structure $(\bar{\mathbf{5}}_1, \bar{\mathbf{5}}_1', \bar{\mathbf{5}}_2)$ is important to obtain the non-vanishing $\Delta m_{\bar{5},2}^2$, this can be a critical signature of the $E_6 \times SU(2)_F \times U(1)_A$ model, especially when $\Delta m_{10,2}^2 = \Delta m_{\bar{5},3}^2$ is vanishing[2].

[2] For example, if we use non-Abelian discrete symmetry instead of the $SU(2)_F$ local family symmetry, $\Delta m_{10,2}^2 = \Delta m_{\bar{5},3}^2 = 0$ because the discrete symmetry has no D-term.

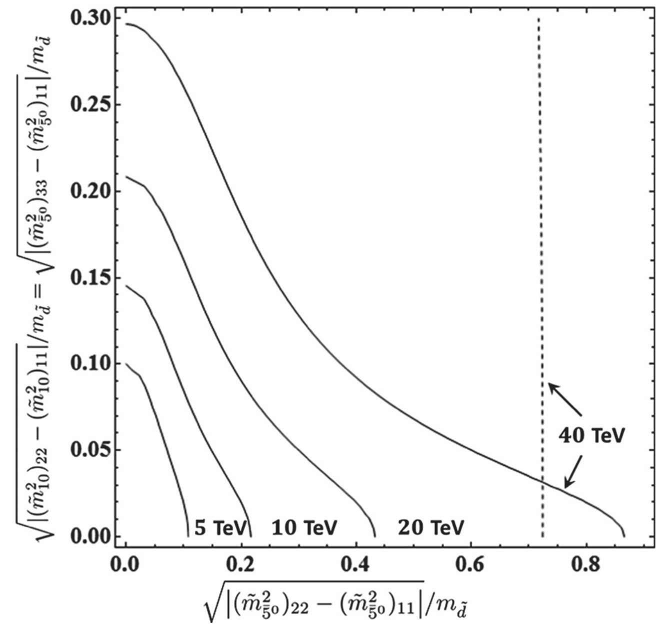

Fig. 1. Allowed region in $\left(\sqrt{|\Delta m^2_{\bar{5},2}|}\Big/m_{\tilde{d}}, \sqrt{|\Delta m^2_{10,2}|}\Big/m_{\tilde{d}} = \sqrt{|\Delta m^2_{\bar{5},3}|}\Big/m_{\tilde{d}}\right)$ space. The allowed region for the condition of $\sqrt{|\mathrm{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}|}$ is obtained below the solid lines for various $m_{\tilde{d}} = 5$ TeV, 10 TeV, 20 TeV, and 40 TeV. The allowed region for $\sqrt{|\mathrm{Im}(\delta^d_{12})^2_{RR}|}$ is the left side of the dotted line for $m_{\tilde{d}} = 40$ TeV. The other conditions are satisfied in the allowed region for $\sqrt{|\mathrm{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}|}$.

In the above arguments, we have neglected the contributions to the sfermion masses that are suppressed by the power of λ in Eqs. (13) and (14). All these terms except the $\lambda^2 m^2$ term in Eq. (14) can be neglected in the above arguments. However, the $\lambda^2 m^2$ term gives non-vanishing $\sqrt{|\Delta m^2_{\bar{5},2}|}\Big/m_{\tilde{d}}$, which becomes $O(\lambda)$ if $m \sim m_0$. The FCNC constraints in this situation can be easily extracted from Fig. 1.

At the end of this section, we comment about the CEDM constraints. As noted in Refs. [44–46], the CEDM constraints from the neutron (Hg) are very severe, especially for the models with natural SUSY-type sfermion masses like the MUSM as

$$\mathrm{Im}\left[(\delta^u_{13})_{LL}(\delta^u_{31})_{RR}\right] < 9.1\times10^{-7}(1.2\times10^{-6})\cdot\left(\frac{m_3}{500\,\mathrm{GeV}}\right)^2. \tag{34}$$

Since we usually take the complex Yukawa couplings to obtain the sizable Kobayashi–Maskawa (KM) phase, $(\delta^u_{13})_{LL}$ and $(\delta^u_{31})_{RR}$ become complex generically. If $(\delta^u_{13})_{LL}(\delta^u_{31})_{RR}$ has $O(1)$ complex phase, the above constraints cannot be satisfied because $|(\delta^u_{13})_{LL}(\delta^u_{31})_{RR}| \sim \lambda^6 \sim 10^{-4}$ in this model. Note that models with the natural SUSY-type sfermion mass spectrum are severely constrained by the CEDM generically. However, in $E_6 \times SU(2)_F \times U(1)_A$ with spontaneous CP violation, the up-type Yukawa matrix becomes real as in Eq. (6) and therefore L_u and R_u are also real as in Eq. (20). As a result, $(\delta^u_{13})_{LL}$ and $(\delta^u_{31})_{RR}$ become real, and the above severe constraints can be satisfied in a non-trivial way.

5. Discussions and summary

We have shown that the sizable D-term contributions to the sfermion mass spectrum can be signatures of a certain GUT, $E_6 \times SU(2)_F \times U(1)_A$ GUT. Note that these D-term contributions destroy the degeneracy of sfermion masses among different generations in this model. This point is one large difference between our work and the previous works, which have argued the D-term contributions [58–61], which destroy the degeneracy of masses only between sfermions with different quantum charges, as a signature of GUT with larger rank unification group. Such D-terms are strongly constrained by the FCNC processes if the SUSY breaking scale is the weak scale. However, in $E_6 \times SU(2)_F \times U(1)_A$, a natural SUSY-type sfermion mass spectrum is obtained, and if the masses of $\mathbf{10}_3$ sfermions are larger than $O(1\,\text{TeV})$ to realize the 126 GeV Higgs and the other sfermion masses are $O(10\,\text{TeV})$, then a sizable D-term contribution is allowed. A novel relation $\tilde{m}^2_{\bar{\mathbf{5}}_3} - \tilde{m}^2_{\bar{\mathbf{5}}_1} = \tilde{m}^2_{\mathbf{10}_2} - \tilde{m}^2_{\mathbf{10}_1}$ is predicted in this model. If these D-terms can be observed in future experiments like the 100 TeV proton collider or muon collider, we may confirm the $E_6 \times SU(2)_F \times U(1)_A$ GUT.

Since we have in mind the generalized mirage mediation scenario in which the mirage scale is the weak scale [50], we have not considered the RG effects in estimating the FCNC constraints in this paper. However, for the other SUSY breaking scenario, we have to consider the renormalization group (RG) effects in the estimation generically. It is possible that m_3 is much smaller than 1 TeV, while a sufficiently large stop mass for the 126 GeV Higgs can be obtained from the RG effects via the gluino. In such a situation, the lepton flavor violation processes can be sizable [28–31]. However, the constraint is quite weak as $m_3 > 200\,\text{GeV}$.

Since the GUT scale is much larger than the TeV scale that we can reach by experiments, it is important to consider how to test the GUT scenario. We have discussed the D-term contributions that are dependent on generations, and they can be a promising signature of the $E_6 \times SU(2)_F \times U(1)_A$ GUT scenario.

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Appendix A. The coefficients of diagonalizing matrices (in leading order)

In Appendix A in Ref. [49], we showed how to diagonalize the 3×3 matrix Y_{ij}. Here we show the diagonalizing matrices for up-type quark, down-type quark, and charged lepton. The diagonalizing matrices L_ψ and R_ψ come from mixing angles $s^{\psi L/R}_{ij} \equiv \sin\theta^{\psi L/R}_{ij} e^{i\chi^{\psi L/R}_{ij}}$ and $c^{\psi L/R}_{ij} \equiv \cos\theta^{\psi L/R}_{ij}$. In our calculation we use the approximation that the mixing angles are small, i.e., $\left|s^{\psi L/R}_{ij}\right| \sim \left|\theta^{\psi L/R}_{ij}\right| \ll 1$ $\left(s^{\psi L/R}_{ij} \sim \theta^{\psi L/R}_{ij} e^{i\chi^{\psi L/R}_{ij}}\right)$ and $c^{\psi L/R}_{ij} \simeq 1$. In this approximation the diagonalizing matrices are

$$L_\psi \simeq \begin{pmatrix} 1 & s^{\psi L*}_{12} & s^{\psi L*}_{13} \\ -s^{\psi L}_{12} & 1 & s^{\psi L*}_{23} \\ -s^{\psi L}_{13} + s^{\psi L}_{23} s^{\psi L}_{12} & -s^{\psi L}_{23} & 1 \end{pmatrix}, \tag{A1}$$

$$R_\psi \simeq \begin{pmatrix} 1 & s_{12}^{\psi R} & s_{13}^{\psi R} \\ -s_{12}^{\psi R*} & 1 & s_{23}^{\psi R} \\ -s_{13}^{\psi R*} + s_{23}^{\psi R*} s_{12}^{\psi R*} & -s_{23}^{\psi R*} & 1 \end{pmatrix}. \tag{A2}$$

From Eq. (6), the mixing angles for the up-type quark are calculated as

$$s_{23}^{uL} = s_{23}^{uR*} \simeq \frac{b}{a}\lambda^2 \equiv R_{23}^{uL}\lambda^2,\ s_{13}^{uL} = s_{13}^{uR*} \simeq 0,\ s_{12}^{uL} = -s_{12}^{uR*} \simeq \frac{\frac{1}{3}ad_q}{ac-b^2}\lambda \equiv \frac{1}{3}R_{12}^{uL}\lambda. \tag{A3}$$

From Eq. (7), the mixing angles for the down-type quark are calculated as

$$s_{23}^{dL} \simeq \frac{cg-bf}{bg-af}\lambda^2 \equiv R_{23}^{dL}\lambda^2, s_{13}^{dL} \simeq \frac{1}{3}\frac{d_q g}{bg-af}\lambda^3 \equiv \frac{1}{3}R_{13}^{dL}\lambda^3, \tag{A4}$$

$$\begin{aligned} s_{12}^{dL} &\simeq -\frac{2}{3}\frac{(bg-af)^2 d_5}{(ac-b^2)\{f(bg-af)-g(cg-bf)\}}\lambda \\ &\quad + \frac{4}{27}\frac{a^2 d_q d_5^2}{(ac-b^2)\{f(bg-af)-g(cg-bf)\}\beta_H}e^{-i(2\rho-\delta)}\lambda \\ &\equiv \left(\frac{2}{3}R_{12}^{dL} + \frac{4}{27}I_{12}^{dL}e^{-i(2\rho-\delta)}\right)\lambda, \end{aligned}$$

$$\begin{aligned} s_{23}^{dR*} &\simeq \frac{g^2\beta_H}{bg-af}e^{i(\rho-\delta)}\lambda^{0.5} - \frac{4}{9}\frac{d_5^2 a^2}{(ac-b^2)(bg-af)}e^{-i\rho}\lambda^{0.5} \\ &\equiv I_{23}^{dR}e^{i(\rho-\delta)}\lambda^{0.5} - \frac{4}{9}I_{23}^{\prime dR}e^{-i\rho}\lambda^{0.5},\ s_{13}^{dR*} \simeq -\frac{2}{3}\frac{ad_5}{ac-b^2}\lambda \equiv \frac{2}{3}R_{13}^{dR}\lambda, \end{aligned} \tag{A5}$$

$$s_{12}^{dR*} \simeq \frac{2}{3}\frac{d_5(bg-af)}{\{f(bg-af)-g(cg-bf)\}\beta_H}e^{-i(\rho-\delta)}\lambda^{0.5} \equiv \frac{2}{3}I_{12}^{dR}e^{-i(\rho-\delta)}\lambda^{0.5}.$$

From Eq. (8), the mixing angles for the charged lepton are calculated as

$$s_{23}^{eL} \simeq \frac{g^2\beta_H}{bg-af}e^{i(\rho-\delta)}\lambda^{0.5} \equiv I_{23}^{dR}e^{i(\rho-\delta)}\lambda^{0.5},\ s_{13}^{eL} \simeq 0, \tag{A6}$$

$$s_{12}^{eL} \simeq \frac{d_l(bg-af)}{\beta_H\{f(bg-af)-g(cg-bf)\}}e^{-i(\rho-\delta)}\lambda^{0.5} \equiv I_{12}^{eL}e^{-i(\rho-\delta)}\lambda^{0.5}$$

$$s_{23}^{eR*} \simeq s_{23}^{dL} \equiv R_{23}^{dL}\lambda^2,\ s_{13}^{eR*} \simeq -\frac{d_l g}{bg-af}\lambda^3 \equiv R_{13}^{eR}\lambda^3, \tag{A7}$$

$$s_{12}^{eR*} \simeq \frac{d_l g^2}{\{f(bg-af)-g(cg-bf)\}}\lambda \equiv R_{12}^{eR}\lambda.$$

The diagonalizing matrices for up-type quark, down-type quark, and charged lepton are calculated as

$$L_u \sim \begin{pmatrix} 1 & \frac{1}{3}R^{uL}_{12}\lambda & 0 \\ -\frac{1}{3}R^{uL}_{12}\lambda & 1 & R^{uL}_{23}\lambda^2 \\ \frac{1}{3}R^{uL}_{23}R^{uL}_{12}\lambda^3 & -R^{uL}_{23}\lambda^2 & 1 \end{pmatrix}, \tag{A8}$$

$$R_u \sim \begin{pmatrix} 1 & -\frac{1}{3}R^{uL}_{12}\lambda & 0 \\ \frac{1}{3}R^{uL}_{12}\lambda & 1 & R^{uL}_{23}\lambda^2 \\ -\frac{1}{3}R^{uL}_{23}R^{uL}_{12}\lambda^3 & -R^{uL}_{23}\lambda^2 & 1 \end{pmatrix}, \tag{A9}$$

$$L_d = \begin{pmatrix} 1 & \left(\frac{2}{3}R^{dL}_{12} + \frac{4}{27}I^{dL}_{12}e^{i(2\rho-\delta)}\right)\lambda & \frac{1}{3}R^{dL}_{13}\lambda^3 \\ -\left(\frac{2}{3}R^{dL}_{12} + \frac{4}{27}I^{dL}_{12}e^{-i(2\rho-\delta)}\right)\lambda & 1 & R^{dL}_{23}\lambda^2 \\ \left(-\frac{1}{3}R^{dL}_{13} + \frac{2}{3}R^{dL}_{23}R^{dL}_{12} + \frac{4}{27}R^{dL}_{23}I^{dL}_{12}e^{-i(2\rho-\delta)}\right)\lambda^3 & -R^{dL}_{23}\lambda^2 & 1 \end{pmatrix}, \tag{A10}$$

$$R_d = \begin{pmatrix} 1 & \frac{2}{3}I^{dR}_{12}e^{i(\rho-\delta)}\lambda^{0.5} & \frac{2}{3}R^{dR}_{13}\lambda \\ -\frac{2}{3}I^{dR}_{12}e^{-i(\rho-\delta)}\lambda^{0.5} & 1 & I^{dR}_{23}e^{-i(\rho-\delta)}\lambda^{0.5} \\ \left(-\frac{2}{3}R^{dR}_{13} + \frac{2}{3}I^{dR}_{23}I^{dR}_{12} - \frac{8}{27}I'^{dR}_{23}I^{dR}_{12}e^{-i(2\rho-\delta)}\right)\lambda & -I^{dR}_{23}e^{i(\rho-\delta)}\lambda^{0.5} & 1 \end{pmatrix}, \tag{A11}$$

$$L_e \sim \begin{pmatrix} 1 & I^{eL}_{12}e^{i(\rho-\delta)}\lambda^{0.5} & 0 \\ -I^{eL}_{12}e^{-i(\rho-\delta)}\lambda^{0.5} & 1 & I^{dR}_{23}e^{-i(\rho-\delta)}\lambda^{0.5} \\ I^{dR}_{23}I^{eL}_{12}\lambda & -I^{dR}_{23}e^{i(\rho-\delta)}\lambda^{0.5} & 1 \end{pmatrix}, \tag{A12}$$

$$R_e \sim \begin{pmatrix} 1 & R^{eR}_{12}\lambda & R^{eR}_{13}\lambda^3 \\ -R^{eR}_{12}\lambda & 1 & R^{dL}_{23}\lambda^2 \\ (-R^{eR}_{13} + R^{dL}_{23}R^{eR}_{12})\lambda^3 & -R^{dL}_{23}\lambda^2 & 1 \end{pmatrix}. \tag{A13}$$

In this model the Majorana neutrino mass matrix has a lot of other real parameters and CP phases, and, therefore, we cannot constrain the diagonalizing matrix for the neutrino. The diagonalizing matrix for the neutrino is written as

$$L_\nu \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}, \tag{A14}$$

where we have omitted the complex $O(1)$ coefficients. In this model we can obtain realistic CKM and MNS matrices as

$$U_{\mathrm{CKM}} = L_u^\dagger L_d \sim \begin{pmatrix} 1 & \left(-\frac{1}{3}R_{12}^{uL} + \frac{2}{3}R_{12}^{dL} + \frac{4}{27}I_{12}^{dL}e^{i(\rho-\delta)}\right)\lambda & O(\lambda^4) \\ \left(\frac{1}{3}R_{12}^{uL} - \frac{2}{3}R_{12}^{dL} - \frac{4}{27}I_{12}^{dL}e^{-i(2\rho-\delta)}\right)\lambda & 1 & \left(-R_{23}^{uL} + R_{23}^{dL}\right)\lambda^2 \\ \left\{-\frac{2}{3}R_{23}^{uL}R_{12}^{dL} - \frac{1}{3}R_{13}^{dL} + \frac{2}{3}I_{23}^{dL}I_{12}^{dL} - \frac{4}{27}(R_{23}^{uL} - R_{23}^{dL})I_{12}^{dL}e^{-i(2\rho-\delta)}\right\}\lambda^3 & \left(R_{23}^{uL} - R_{23}^{dL}\right)\lambda^2 & 1 \end{pmatrix}, \tag{A15}$$

$$\left|U_{\mathrm{MNS}}\right| = \left|L_\nu^\dagger L_e\right| \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}. \tag{A16}$$

As discussed in Refs. [44–47], the leading contribution to the component $(U_{\mathrm{CKM}})_{13}$ is canceled and the sub-leading contribution $O(\lambda^4)$ dominates $(U_{\mathrm{CKM}})_{13}$.

Appendix B. Mass insertions

In this appendix, we just show all the mass insertion parameters in this model.

$$(\delta_{12}^u)_{LL} = -(\delta_{12}^u)_{RR} \simeq \left\{-\frac{1}{3}R_{12}^{uL}\lambda\Delta m_{10,2}^2 - \frac{1}{3}(R_{23}^{uL})^2 R_{12}^{uL}\lambda^5\Delta m_{10,3}^2\right\}\Big/ m_{\tilde{u}}^2 \tag{B1}$$

$$(\delta_{13}^u)_{LL} = -(\delta_{13}^u)_{RR} \simeq \left\{-\frac{1}{3}R_{23}^{uL}R_{12}^{uL}\Delta m_{10,2}^2 + \frac{1}{3}R_{23}^{uL}R_{12}^{uL}\Delta m_{10,3}^2\right\}\lambda^3\Big/ m_{\tilde{u}}^2 \tag{B2}$$

$$(\delta_{23}^u)_{LL} = (\delta_{23}^u)_{RR} \simeq R_{23}^{uL}\{\Delta m_{10,2}^2 - \Delta m_{10,3}^2\}\lambda^2/m_{\tilde{u}}^2 \tag{B3}$$

$$\begin{aligned}(\delta_{12}^d)_{LL} \simeq &\left\{-\left(\frac{2}{3}R_{12}^{dL} + \frac{4}{27}I_{12}^{dL}e^{i(2\rho-\delta)}\right)\lambda\Delta m_{10,2}^2\right.\\ &\left.-R_{23}^{dL}\left(-\frac{1}{3}R_{13}^{dL} + \frac{2}{3}R_{23}^{dL}R_{12}^{dL} + \frac{4}{27}R_{23}^{dL}I_{12}^{dL}e^{i(2\rho-\delta)}\right)\lambda^5\Delta m_{10,3}^2\right\}\Big/ m_{\tilde{d}}^2\end{aligned} \tag{B4}$$

$$\begin{aligned}(\delta_{13}^d)_{LL} \simeq &\left\{-R_{23}^{dL}\left(\frac{2}{3}R_{12}^{dL} + \frac{4}{27}I_{12}^{dL}e^{i(2\rho-\delta)}\right)\Delta m_{10,2}^2\right.\\ &\left.+\left(-\frac{1}{3}R_{13}^{dL} + \frac{2}{3}R_{23}^{dL}R_{12}^{dL} + \frac{4}{27}R_{23}^{dL}I_{12}^{dL}e^{i(2\rho-\delta)}\right)\Delta m_{10,3}^2\right\}\lambda^3\Big/ m_{\tilde{d}}^2\end{aligned} \tag{B5}$$

$$(\delta_{23}^d)_{LL} \simeq R_{23}^{dL}\{\Delta m_{10,2}^2 - \Delta m_{10,3}^2\}\lambda^2/m_{\tilde{d}}^2 \tag{B6}$$

$$(\delta_{12}^d)_{RR} \simeq \left\{-\frac{2}{3}I_{12}^{dR}e^{i(\rho-\delta)}\lambda^{0.5}\Delta m_{\bar{5},2}^2 - I_{23}^{dR}\left(-\frac{2}{3}R_{13}^{dR} + \frac{2}{3}I_{23}^{dR}I_{12}^{dR}\right)e^{i(\rho-\delta)}\lambda^{1.5}\Delta m_{\bar{5},3}^2\right\}\Big/ m_{\tilde{d}}^2 \tag{B7}$$

$$\begin{aligned}(\delta_{13}^d)_{RR} \simeq &\left\{\left(-\frac{2}{3}I_{23}^{dR}I_{12}^{dR} + \frac{8}{27}I_{12}^{dR}I_{23}^{\prime dR}e^{i(2\rho-\delta)}\right)\Delta m_{\bar{5},2}^2\right.\\ &\left.+\left(-\frac{2}{3}R_{13}^{dR} + \frac{2}{3}I_{23}^{dR}I_{12}^{dR} - \frac{8}{27}I_{23}^{\prime dR}I_{12}^{dR}e^{i(2\rho-\delta)}\right)\Delta m_{\bar{5},3}^2\right\}\lambda\Big/ m_{\tilde{d}}^2\end{aligned} \tag{B8}$$

$$(\delta^d_{23})_{RR} \simeq I^{dR}_{23} e^{-i(\rho-\delta)} \{\Delta m^2_{\bar{5},2} - \Delta m^2_{\bar{5},3}\} \lambda^{0.5} / m^2_{\tilde{d}} \tag{B9}$$

$$(\delta^e_{12})_{LL} \simeq -I^{eL}_{12} e^{i(\rho-\delta)} \{\lambda^{0.5} \Delta m^2_{\bar{5},2} + (I^{dR}_{23})^2 \lambda^{1.5} \Delta m^2_{\bar{5},3}\} / m^2_{\tilde{e}} \tag{B10}$$

$$(\delta^e_{13})_{LL} \simeq -I^{dR}_{23} I^{eL}_{12} \{\Delta m^2_{\bar{5},2} - \Delta m^2_{\bar{5},3}\} \lambda / m^2_{\tilde{e}} \tag{B11}$$

$$(\delta^e_{23})_{LL} \simeq I^{dR}_{23} e^{-i(\rho-\delta)} \{\Delta m^2_{\bar{5},2} - \Delta m^2_{\bar{5},3}\} \lambda^{0.5} / m^2_{\tilde{e}} \tag{B12}$$

$$(\delta^e_{12})_{RR} \simeq \{ - R^{eR}_{12} \lambda \Delta m^2_{10,2} - R^{dL}_{23} (- R^{eR}_{13} + R^{dL}_{23} R^{eR}_{12}) \lambda^5 \Delta m^2_{10,3}\} / m^2_{\tilde{e}} \tag{B13}$$

$$(\delta^e_{13})_{RR} \simeq \{ - R^{dL}_{23} R^{eR}_{12} \Delta m^2_{10,2} + (- R^{eR}_{13} + R^{dL}_{23} R^{eR}_{12}) \Delta m^2_{10,3}\} \lambda^3 / m^2_{\tilde{e}} \tag{B14}$$

$$(\delta^e_{23})_{RR} \simeq R^{dL}_{23} \{\Delta m^2_{10,2} - \Delta m^2_{10,3}\} \lambda^2 / m^2_{\tilde{e}} \tag{B15}$$

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